

Microscopic optical potentials for ^4He scattering

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We present a reliable double-folding (DF) model for ^4He -nucleus scattering, using the Melbourne g -matrix nucleon-nucleon interaction that explains nucleon-nucleus scattering with no adjustable parameter. In the DF model, only the target density is taken as the local density in the Melbourne g -matrix. For ^4He elastic scattering from ^{58}Ni and ^{208}Pb targets in a wide range of incident energies from 20 MeV/nucleon to 200 MeV/nucleon, the DF model with the target-density approximation (TDA) yields much better agreement with the experimental data than the usual DF model with the frozen-density approximation in which the sum of projectile and target densities is taken as the local density. We also discuss the relation between the DF model with the TDA and the conventional folding model in which the nucleon-nucleus potential is folded with the ^4He density.

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I. INTRODUCTION

Microscopic derivation of nucleon-nucleus (NA) and nucleus-nucleus (AA) optical potentials is a goal of nuclear reaction theory. The optical potential is an important quantity to describe not only the elastic scattering but also more complicated reactions such as inelastic scattering, breakup and transfer reactions. For the latter case, the optical potential is used as a key input in theoretical calculations such as the distorted-wave Born approximation and the continuum discretized coupled-channels method [1–3].

The g -matrix folding model is a powerful tool of deriving NA and AA optical potentials. In the model, the optical potential is calculated by folding the g -matrix effective nucleon-nucleon (NN) interaction [4–13] with the target density for NA scattering and the projectile and target densities for AA scattering; see for example Refs. [14–18] for the folding procedure. The folding model for NA and AA scattering are referred to as the single-folding model and the double-folding (DF) model, respectively. For NA elastic scattering, the model is quite successful in reproducing the experimental data systematically with no free parameter, when the Melbourne g -matrix [11] is used as an effective NN interaction in the folding calculations. As an important advantage of the g -matrix folding model, the model takes account of nuclear medium effects. The g -matrix is calculated in nuclear matter and hence depends on nuclear-matter density ρ . When the optical potential is evaluated from the g -matrix in the folding procedure, the nuclear-matter density is replaced by the target density at the location of interacting nu-

cleon pair. This approximation is called the local-density approximation.

The NA potential thus derived is non-local and thereby not so practical in many applications. It is, however, possible to localize the potential with the Brieva-Rook approximation [6]. Recently the validity of the approximation was shown in Ref. [19, 20]. In fact, the local version of g -matrix folding potential describes NA scattering with no adjustable parameter [21], and close to the phenomenological NA optical potentials [22–25].

From a theoretical viewpoint based on the multiple scattering theory [26–28], the multiple NN scattering series in AA collision [28] is more complicated than that in NA collision [26, 27]. In this sense, microscopic understanding of the optical potentials is relatively more difficult for AA scattering than for NA scattering. One of the simplest composite projectiles is ^4He , since it is almost inert. For ^4He -nucleus elastic scattering, a systematic analysis was made [29] by using the g -matrix interaction proposed by Jeukenne, Lejeune, and Mahaux (JLM) [5]. The JLM g -matrix folding model reproduces measured differential cross sections for ^4He elastic scattering at incident energies ranging from 10 to 60 MeV/nucleon, if the real and imaginary parts of the folding potential are reduced by about 25% and 35%, respectively. In the JLM g -matrix, nuclear medium effects are included only partly, so that the normalization factors are always introduced. This fact strongly suggests that the parameter-free analysis based on the Melbourne g -matrix folding model should be made for ^4He -nucleus elastic scattering.

In the DF procedure, the frozen-density approximation (FDA) is usually taken. Namely, one takes as the local density the sum of projectile and target densities, ρ_P and ρ_T , at the midpoint of interacting two nucleons, one in projectile (P) and the other in target (T):

$$g(\rho) = g(\rho_P + \rho_T). \quad (1)$$

Very recently, the Melbourne g -matrix folding model with the FDA was applied to $^{12}\text{C}+^{12}\text{C}$ and $^{20-32}\text{Ne}+^{12}\text{C}$ elastic scattering at intermediate energies with success in

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reproducing measured differential cross sections $d\sigma/d\Omega$ and total reaction cross sections σ_R with no free parameter [18, 30, 31]. In the calculations, the densities of unstable nuclei $^{20-32}\text{Ne}$ were evaluated by antisymmetrized molecular dynamics (AMD) [32, 33] with the Gogny-D1S interaction [34]. The AMD wave functions successfully describe low-lying spectra of Ne isotopes [32]. The microscopic approach concluded that $^{30-32}\text{Ne}$ in the “island of inversion” have large deformation and ^{31}Ne has a deformed halo structure [18, 30, 31]. This indicates that the $N = 20$ magicity disappears. The Melbourne g -matrix folding model is thus a powerful tool of not only understanding the reaction mechanism but also determining the structure of unstable nuclei.

In this paper, we microscopically describe ^4He elastic scattering from heavier targets such as ^{58}Ni and ^{208}Pb in a wide range of incident energies from 20 MeV/nucleon to 200 MeV/nucleon, using the Melbourne g -matrix DF model with no adjustable parameter. After showing that the DF model with the FDA cannot reproduce measured $d\sigma/d\Omega$ and σ_R for the scattering, we propose a new approximation instead of the FDA. In the approximation, only the target density is taken as the local density. This approximation is referred to as the target-density approximation (TDA) in this paper. The reliability of the TDA is shown theoretically with the multiple scattering theory [26–28] and phenomenologically by showing that the DF model with the TDA well reproduces the data on $d\sigma/d\Omega$ and σ_R . We also investigate the reliability of the conventional nucleon-nucleus folding (NAF) model in which the NA potential is folded with the ^4He density.

In Sec. II, we recapitulate the Melbourne g -matrix DF model and show the reliability of the TDA theoretically. Numerical results are shown in Sec. III. Section IV is devoted to a summary.

II. MODEL BUILDING

AA scattering can be described by the many-body Schrödinger equation,

$$(T_R + h_P + h_T + \sum_{i \in P, j \in T} v_{ij} - E)\Psi^{(+)} = 0, \quad (2)$$

with the realistic NN interaction v_{ij} , where T_R stands for the kinetic energy with respect to the relative coordinate (\mathbf{R}) between the projectile (P) and the target (T), E is the total energy and h_P (h_T) means the internal Hamiltonian of P (T). Using the multiple scattering theory [26, 27] for AA scattering [28], one can rewrite Eq. (2) into

$$(T_R + h_P + h_T + \sum_{i \in P, j \in T} \tau_{ij} - E)\hat{\Psi}^{(+)} = 0 \quad (3)$$

with the effective NN interaction τ_{ij} defined by

$$\tau_{ij} = v_{ij} + v_{ij}G_0\tau_{ij} \quad (4)$$

with

$$G_0 = \frac{\mathcal{P}_P \mathcal{P}_T}{E - K - h_P - h_T + i\epsilon}, \quad (5)$$

where \mathcal{P}_P (\mathcal{P}_T) denotes the projection operator onto the space of antisymmetrized wave functions of P (T). In the derivation of Eq. (3), the antisymmetrization between nucleons in P and those in T has been neglected, but it is shown in Refs. [35, 36] that the antisymmetrization effects are well taken care of by using τ_{ij} that is properly symmetrical with respect to the exchange of the colliding nucleons. Since the effective NN interaction τ_{ij} includes nuclear medium effects, the g -matrix (g_{ij}) is often used as such τ_{ij} [4–12, 16].

Since g_{ij} also includes projectile- and target-excitation effects approximately, Eq. (3) can be further rewritten into the single-channel equation

$$[T_R + U - E_{\text{in}}]\psi = 0, \quad (6)$$

with the folding potential

$$U(\mathbf{R}) = \langle \Phi_0 | \sum_{i \in P, j \in T} g_{ij} | \Phi_0 \rangle, \quad (7)$$

where the incident energy E_{in} is related to the total energy E as $E = E_{\text{in}} + e_0(P) + e_0(T)$ for the grand-state energies, $e_0(P)$ and $e_0(T)$, of P and T. The wave function Φ_0 denotes the product of the ground states of P and T, while ψ means the relative wave function between P and T. This is nothing but the g -matrix DF model. In the actual calculations, the FDA shown in Eq. (1) is usually taken and the Coulomb potential U_{Coul} is added to the resulting U .

Now we consider ^4He scattering from heavier nuclei. In the scattering, the projectile (^4He) is hardly excited, whereas the target is excited easily. As a good approximation we can hence neglect projectile excitations. Namely, we can replace h_P by the ground-state energy $e_0(P)$ and hence $\mathcal{P}_P \mathcal{P}_T$ by \mathcal{P}_T :

$$G_0 \approx \frac{\mathcal{P}_T}{E - K - e_0(P) - h_T + i\epsilon}. \quad (8)$$

After the approximation, the τ_{ij} includes nuclear medium effects from T, but not from P. We should therefore replace τ_{ij} by the g -matrix depending only on ρ_T :

$$g(\rho) = g(\rho_T). \quad (9)$$

This is the TDA proposed in the present paper. The reliability of the TDA is confirmed also phenomenologically in Sec. III by comparing the theoretical results with the experimental data and showing that the TDA is much better than the FDA.

Next we recapitulate the single folding model for NA scattering and the DF model for AA scattering. As for the detail of the models, for example, see Refs. [6, 12, 13, 15, 19, 21] for NA scattering and Refs. [12, 14, 17, 18, 20]

for AA scattering. The DF potential $U = V + iW$ consists of the direct and exchange parts, U^{DR} and U^{EX} [14, 17]:

$$U(\mathbf{R}) = U^{\text{DR}}(\mathbf{R}) + U^{\text{EX}}(\mathbf{R}) + U_{\text{Coul}}(\mathbf{R}) \quad (10)$$

with

$$U^{\text{DR}}(\mathbf{R}) = \sum_{\mu, \nu} \int \rho_{\text{P}}^{\mu}(\mathbf{r}_{\text{P}}) \rho_{\text{T}}^{\nu}(\mathbf{r}_{\text{T}}) g_{\mu\nu}^{\text{DR}}(s; \rho_{\mu\nu}) d\mathbf{r}_{\text{P}} d\mathbf{r}_{\text{T}}, \quad (11)$$

$$U^{\text{EX}}(\mathbf{R}) = \sum_{\mu, \nu} \int \rho_{\text{P}}^{\mu}(\mathbf{r}_{\text{P}}, \mathbf{r}_{\text{P}} - \mathbf{s}) \rho_{\text{T}}^{\nu}(\mathbf{r}_{\text{T}}, \mathbf{r}_{\text{T}} + \mathbf{s}) \times g_{\mu\nu}^{\text{EX}}(s; \rho_{\mu\nu}) \exp[-i\mathbf{K}(\mathbf{R}) \cdot \mathbf{s}/M] d\mathbf{r}_{\text{P}} d\mathbf{r}_{\text{T}}, \quad (12)$$

where \mathbf{r}_{P} (\mathbf{r}_{T}) stands for the coordinate of the interacting nucleon from the center of mass of P (T), $\mathbf{s} = \mathbf{r}_{\text{P}} - \mathbf{r}_{\text{T}} + \mathbf{R}$, and each of μ and ν denotes the z -component of isospin. Here $\rho_{\text{P}}^{\mu}(\mathbf{r}_{\text{P}})$ and $\rho_{\text{T}}^{\nu}(\mathbf{r}_{\text{T}})$ are one-body densities of P and T and $\rho_{\text{P}}^{\mu}(\mathbf{r}_{\text{P}}, \mathbf{r}_{\text{P}} - \mathbf{s})$ and $\rho_{\text{T}}^{\nu}(\mathbf{r}_{\text{T}}, \mathbf{r}_{\text{T}} + \mathbf{s})$ are mixed densities of P and T, respectively. The non-local U^{EX} has been localized in Eq. (12) with the local semi-classical approximation [6], where the local momentum $\hbar\mathbf{K}(\mathbf{R})$ of P relative to T is defined by $\hbar K(R) \equiv \sqrt{2\mu_{\text{PT}}(E_{\text{in}} - U(R))}$ with the reduced mass μ_{PT} between P and T, and $M = A_{\text{P}}A_{\text{T}}/(A_{\text{P}} + A_{\text{T}})$ for the mass numbers, A_{P} and A_{T} , of P and T. The validity of the localization is shown in Refs. [19, 20]. The direct and exchange parts, $g_{\mu\nu}^{\text{DR}}$ and $g_{\mu\nu}^{\text{EX}}$, of the g -matrix depend on the local density at the midpoint of the interacting nucleon pair:

$$\rho_{\mu\nu} = \rho_{\text{P}}^{\mu}(\mathbf{r}_{\text{P}} - \mathbf{s}/2) + \rho_{\text{T}}^{\nu}(\mathbf{r}_{\text{T}} + \mathbf{s}/2) \quad (13)$$

in the FDA and

$$\rho_{\mu\nu} = \rho_{\text{T}}^{\nu}(\mathbf{r}_{\text{T}} + \mathbf{s}/2) \quad (14)$$

in the TDA; see Ref. [18] for the explicit forms of $g_{\mu\nu}^{\text{DR}}$ and $g_{\mu\nu}^{\text{EX}}$.

We now consider NA scattering at an incident energy E_{in}^{N} . The single folding potentials $U_{\mu} = V_{\mu} + iW_{\mu}$ for proton ($\mu = -1/2$) and neutron ($\mu = 1/2$) scattering are also composed of U_{μ}^{DR} and U_{μ}^{EX} :

$$U_{\mu}(\mathbf{r}_{\mu}) = U_{\mu}^{\text{DR}}(\mathbf{r}_{\mu}) + U_{\mu}^{\text{EX}}(\mathbf{r}_{\mu}) + U_{\text{Coul}}(\mathbf{r}_{\mu}) \quad (15)$$

with

$$U_{\mu}^{\text{DR}}(\mathbf{r}_{\mu}) = \sum_{\nu} \int \rho_{\text{T}}^{\nu}(\mathbf{r}_{\text{T}}) g_{\mu\nu}^{\text{DR}}(s; \rho_{\mu\nu}) d\mathbf{r}_{\text{T}}, \quad (16)$$

$$U_{\mu}^{\text{EX}}(\mathbf{r}_{\mu}) = \sum_{\nu} \int \rho_{\text{T}}^{\nu}(\mathbf{r}_{\text{T}}, \mathbf{r}_{\text{T}} + \mathbf{s}) \times g_{\mu\nu}^{\text{EX}}(s; \rho_{\mu\nu}) \exp[-i\mathbf{K}_{\mu}(\mathbf{r}_{\mu}) \cdot \mathbf{s}] d\mathbf{r}_{\text{T}}, \quad (17)$$

where $\mathbf{s} = \mathbf{r}_{\mu} - \mathbf{r}_{\text{T}}$ for \mathbf{r}_{μ} the coordinate of an incident nucleon from the center of mass of T, the local density $\rho_{\mu\nu}$ is obtained by Eq. (14), and the local momentum $\hbar\mathbf{K}_{\mu}(\mathbf{r}_{\mu})$ between the incident nucleon (N) and T is defined by $\hbar K_{\mu}(r_{\mu}) \equiv \sqrt{2\mu_{\text{NT}}(E_{\text{in}}^{\text{N}} - U_{\mu}(r_{\mu}))}$ for the reduced mass μ_{NT} between N and T.

When AA scattering at high E_{in} is compared with NA scattering at $E_{\text{in}}^{\text{N}} = E_{\text{in}}/A_{\text{P}}$ for heavy targets satisfying $A_{\text{T}} \gg A_{\text{P}} > 1$, the local momenta $\hbar\mathbf{K}_{\mu}(\mathbf{r}_{\mu})$ and $\hbar\mathbf{K}(\mathbf{R})$ nearly agree with their asymptotic values $\hbar\mathbf{K}_{\mu}(\infty)$ and $\hbar\mathbf{K}(\infty)$, respectively, so that

$$\mathbf{K}_{\mu}(\infty) = \mathbf{K}(\infty)/M. \quad (18)$$

Taking the relation (18) and the TDA, one can get

$$U^{\text{DR}}(\mathbf{R}) \approx \sum_{\mu} \int \rho_{\text{P}}^{\mu}(\mathbf{r}_{\text{P}}) U_{\mu}^{\text{DR}}(\mathbf{R} + \mathbf{r}_{\text{P}}) d\mathbf{r}_{\text{P}}, \quad (19)$$

$$U^{\text{EX}}(\mathbf{R}) \approx \sum_{\mu} \int \rho_{\text{P}}^{\mu}(\mathbf{r}_{\text{P}}) U_{\mu}^{\text{EX}}(\mathbf{R} + \mathbf{r}_{\text{P}}) d\mathbf{r}_{\text{P}}. \quad (20)$$

In the derivation of Eq. (20), we have also used the approximation $\rho_{\text{P}}^{\mu}(\mathbf{r}_{\text{P}}, \mathbf{r}_{\text{P}} - \mathbf{s}) \approx \rho_{\text{P}}^{\mu}(\mathbf{r}_{\text{P}}, \mathbf{r}_{\text{P}}) = \rho_{\text{P}}^{\mu}(\mathbf{r}_{\text{P}})$ good in the peripheral region of T that is important for the elastic scattering [19]. For ${}^4\text{He}$ scattering from heavier targets at high E_{in} , the DF potential $U^{\text{DR}} + U^{\text{EX}}$ with the TDA is thus obtained with reasonable accuracy by folding the nucleon-nucleus potential $U_{\mu}^{\text{DR}} + U_{\mu}^{\text{EX}}$ with the projectile density ρ_{P}^{μ} . This is the NAF model mentioned in Sec. I. This model is quite practical, since one can use the phenomenological NA optical potential instead of U_{μ} . The validity of this model is also investigated later in Sec. III. The condition that the local momenta $\hbar\mathbf{K}_{\mu}(\mathbf{r}_{\mu})$ and $\hbar\mathbf{K}(\mathbf{R})$ are close to their asymptotic values is well satisfied at large R , even if E_{in} is small. Since ${}^4\text{He}$ scattering from heavy targets are quite peripheral at small E_{in} , one can expect that the NAF model is good also for small E_{in} . This is also discussed in Sec. III.

III. RESULTS

We analyze measured $d\sigma/d\Omega$ and σ_{R} for ${}^4\text{He}$ elastic scattering from ${}^{58}\text{Ni}$ and ${}^{208}\text{Pb}$ targets in the region $20 \lesssim E_{\text{in}}/A_{\text{P}} \lesssim 200$ MeV, using the following three models:

1. The DF model with the TDA (the DF-TDA model)
2. The DF model with the FDA (the DF-FDA model)
3. The NAF model

As the ${}^4\text{He}$ density ρ_{P} , we use the phenomenological proton-density [37] determined from the electron scattering in which the finite-size effect due to the proton charge is unfolded in the standard manner [38]. The neutron density is assumed to have the same geometry as the proton one. As the target density ρ_{T} , we take the matter densities calculated by the spherical Hartree-Fock (HF) model with the Gogny-D1S interaction [34] in which the spurious center-of-mass motion is removed in the standard manner [18].

Figure 1 shows $d\sigma/d\Omega$ as a function of transfer momentum q for ${}^4\text{He} + {}^{58}\text{Ni}$ scattering in $E_{\text{in}}/A_{\text{P}} = 20\text{--}175$ MeV. For lower incident energies of $E_{\text{in}}/A_{\text{P}} = 20\text{--}60$ MeV,

the DF-FDA model (dotted line) overestimates the experimental data [39–42], but this problem is solved by the DF-TDA model (solid line) that well reproduces the data. For higher energies around $E_{\text{in}}/A_{\text{P}} = 175$ MeV, meanwhile, the DF-FDA model underestimates the experimental data [43], but this problem is also solved by the DF-TDA model that reproduces the data. For intermediate energies of $E_{\text{in}}/A_{\text{P}} = 72$ –120 MeV, the difference between the DF-TDA and DF-FDA results is rather small, so that both the models reasonably reproduce the data. In great detail, for $E_{\text{in}}/A_{\text{P}} = 85$ MeV, the DF-TDA result is better than the DF-FDA result at $q \lesssim 2 \text{ fm}^{-1}$, whereas the latter is better than the former at $q \gtrsim 3 \text{ fm}^{-1}$. For $E_{\text{in}}/A_{\text{P}} = 97$ MeV, the DF-FDA model is slightly better than the DF-TDA model, but this seems to be accidental, since for σ_{R} the DF-TDA model (circles) yields better agreement with the data [43, 45] than the DF-FDA model (squares) as shown in Fig. 2. Throughout these analyses, we can conclude that the DF-TDA model is much better than the DF-FDA model.

Next we compare the DF-TDA model with the NAF model in Figs. 1 and 2. For σ_{R} , the NAF model (triangles) well simulates the DF-TDA result (circles) and hence yields much better agreement with the data [43, 45] than the DF-FDA model (squares). For $d\sigma/d\Omega$ at higher energies of $E_{\text{in}}/A_{\text{P}} = 120$ –175 MeV, as expected, the NAF results (dashed lines) well reproduce the DF-TDA results (solid lines). Also for lower energies of $E_{\text{in}}/A_{\text{P}} = 20$ –43 MeV, the NAF model well simulates the DF-TDA results, since the elastic scattering is quite peripheral, as shown below. For intermediate energies of $E_{\text{in}}/A_{\text{P}} = 60$ –97 MeV, however, the NAF results are deviated sizably from the DF-TDA results.

Figure 3 shows the absolute value of the elastic S -matrix element as a function of R for ${}^4\text{He}+{}^{58}\text{Ni}$ elastic scattering, where R is estimated from the angular momentum L between P and T with the semi-classical relation $L = RK(\infty)$. The solid, dashed and dotted lines correspond to the elastic S -matrix elements calculated with the DF-TDA model at $E_{\text{in}}/A_{\text{P}} = 26, 85$ and 175 MeV, respectively. The ${}^4\text{He}$ scattering becomes more peripheral as $E_{\text{in}}/A_{\text{P}}$ decreases. Particularly at $E_{\text{in}}/A_{\text{P}} = 26$ MeV, the scattering is quite peripheral. This is the reason why the NAF model well simulates the DF-TDA model for lower energies of $E_{\text{in}}/A_{\text{P}} = 20$ –43 MeV. Eventually, the NAF model is good not only for higher energies of $E_{\text{in}}/A_{\text{P}} = 120$ –175 MeV but also for lower energies of $E_{\text{in}}/A_{\text{P}} = 20$ –43 MeV.

Figure 4 shows the folding potentials $U = V + iW$ for ${}^4\text{He}+{}^{58}\text{Ni}$ elastic scattering at $E_{\text{in}}/A_{\text{P}} = 175$ MeV. The FDA has stronger Pauli-blocking effects than the TDA because of $\rho_{\text{P}} + \rho_{\text{T}} \geq \rho_{\text{P}}$. As a result of this property, the DF-FDA potential (dotted line) is less attractive and less absorptive than the DF-TDA potential (solid line). The NAF model (dashed line) well simulates the DF-TDA potential in $R \gtrsim 5$ fm, as expected. This is the reason why at $E_{\text{in}}/A_{\text{P}} = 175$ MeV the NAF model well simulates the DF-TDA model for both $d\sigma/d\Omega$ and σ_{R} .

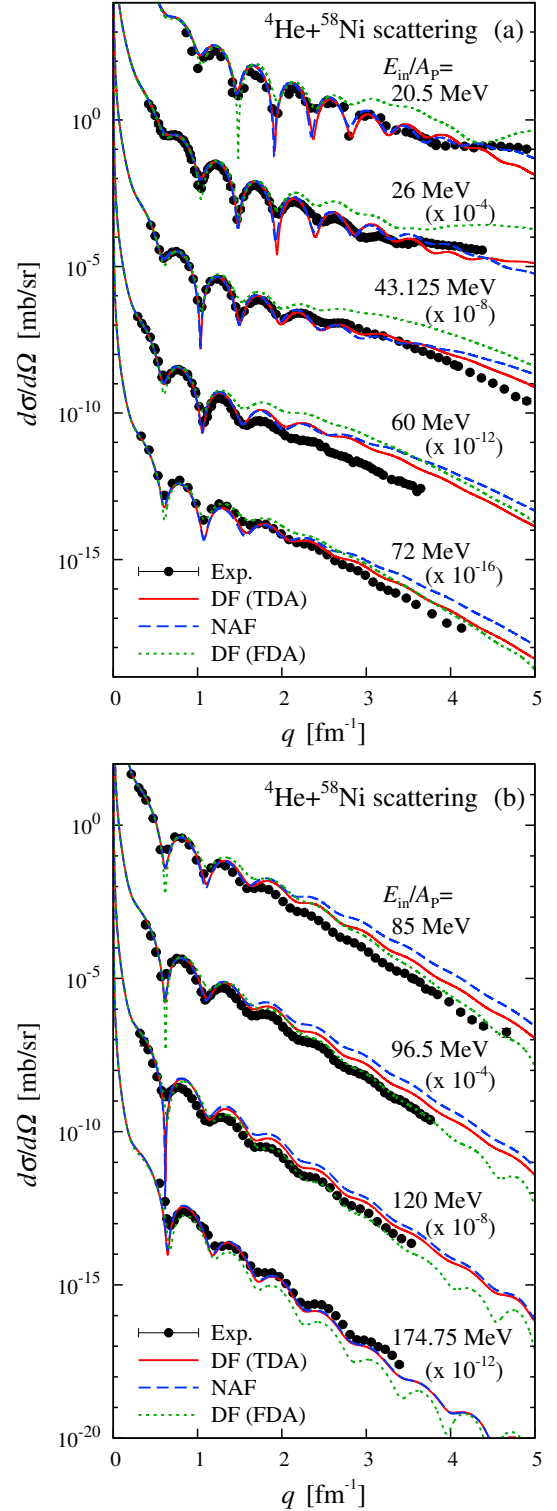


FIG. 1: (Color online) Differential cross sections as a function of transfer momentum q for ${}^4\text{He}+{}^{58}\text{Ni}$ elastic scattering at (a) $E_{\text{in}}/A_{\text{P}} = 20$ –72 MeV and (b) $E_{\text{in}}/A_{\text{P}} = 85$ –175 MeV. The cross section at each $E_{\text{in}}/A_{\text{P}}$ is multiplied by the factor shown in the panel. The solid (dotted) line stands the results of the DF-TDA (DF-FDA) model, whereas the dashed line denotes the results of the NAF model. The experimental data are taken from Refs. [39–44].

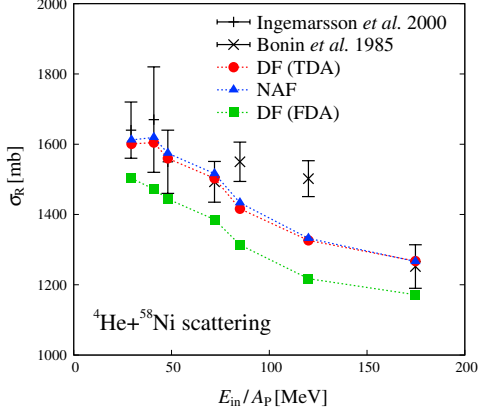


FIG. 2: (Color online) Total reaction cross section σ_R as a function of E_{in}/A_P for ${}^4\text{He}+{}^{58}\text{Ni}$ scattering at $E_{in}/A_P = 20\text{--}175$ MeV. The circles (squares) stand the results of the DF-TDA (DF-FDA) model, whereas the triangles denote the results of the NAF model. The experimental data are taken from [43, 45].

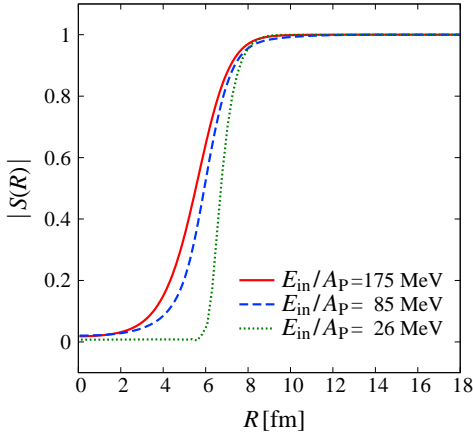


FIG. 3: (Color online) R dependence of the absolute value of the elastic S -matrix element for ${}^4\text{He}+{}^{58}\text{Ni}$ elastic scattering at $E_{in}/A_P = 26, 85$ and 175 MeV. The solid, dashed and dotted lines represent the elastic S -matrix elements calculated with the DF-TDA model at $E_{in}/A_P = 26, 85$ and 175 MeV, respectively.

Figures 5 and 6 show the folding potentials for ${}^4\text{He}+{}^{58}\text{Ni}$ elastic scattering at $E_{in}/A_P = 85$ and 26 MeV, respectively. The Pauli-blocking effects due to ρ_P , which is represented by the difference between the DF-TDA and DF-FDA potentials, become large as E_{in}/A_P decreases, as expected. For $E_{in}/A_P = 26$ MeV, the NAF potential reproduces the DF-TDA potential in $R \gtrsim 5$ fm, but the former largely deviates from the latter in $R \lesssim 5$ fm. The deviation does not contribute to $d\sigma/d\Omega$ and σ_R , since the elastic S -matrix elements are quite small in $R \lesssim 5$ fm. This is the reason why the NAF model is good for lower energies. For $E_{in}/A_P = 85$ MeV, meanwhile, the NAF

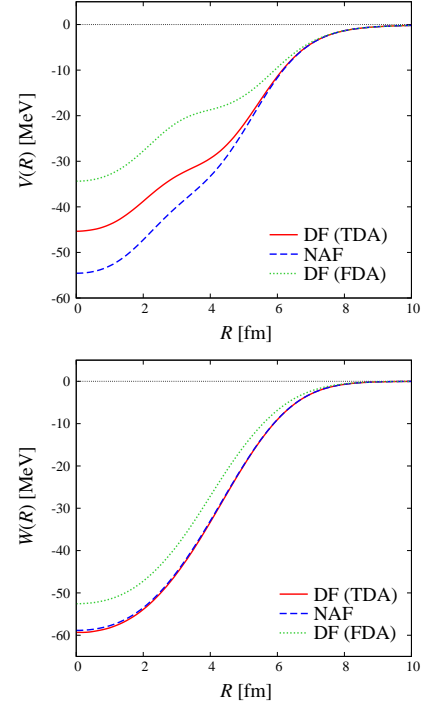


FIG. 4: (Color online) Optical potentials for ${}^4\text{He}+{}^{58}\text{Ni}$ elastic scattering at $E_{in}/A_P = 175$ MeV. The solid (dotted) line stands for the DF-TDA (DF-FDA) potential, whereas the dashed line denotes the NAF potential.

potential is largely deviated from the DF-TDA potential in $R \lesssim 5$ fm, whereas the elastic S -matrix elements are small only in $R \lesssim 3$ fm. The NAF model is thus not good for intermediate energies around $E_{in}/A_P = 85$ MeV.

Finally we briefly discuss ${}^4\text{He}+{}^{208}\text{Pb}$ elastic scattering. Figure 7 shows $d\sigma/d\Omega$ as a function of q for ${}^4\text{He}+{}^{208}\text{Pb}$ scattering in (a) $E_{in}/A_P = 26\text{--}85$ MeV and (b) $E_{in}/A_P = 97\text{--}175$ MeV. The same statement is possible also for ${}^{208}\text{Pb}$ target. Namely, the DF-TDA model yields better agreement with the experimental data [43, 46–48] than the DF-FDA model. The NAF model well simulates the DF-TDA model for lower energies around $E_{in}/A_P = 30$ MeV and also for higher energies around $E_{in}/A_P = 175$ MeV.

IV. SUMMARY

We presented a reliable double-folding (DF) model for ${}^4\text{He}$ scattering from heavier targets such as ${}^{58}\text{Ni}$ and ${}^{208}\text{Pb}$ in a wide range of incident energies from 20 to 200 MeV/nucleon. It is the Melbourne g -matrix DF model with the target-density approximation (TDA), i.e., the DF-TDA model. The reliability of the DF-TDA model was shown theoretically with the multiple scattering theory and phenomenologically by showing that the model reproduces measured $d\sigma/d\Omega$ and σ_R . The

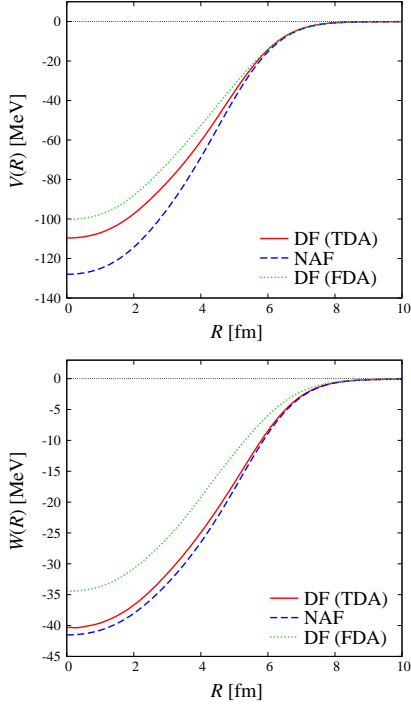


FIG. 5: (Color online) Optical potentials for ${}^4\text{He}+{}^{58}\text{Ni}$ elastic scattering at $E_{\text{in}}/A_P = 85$ MeV. See Fig. 4 for the definition of lines.

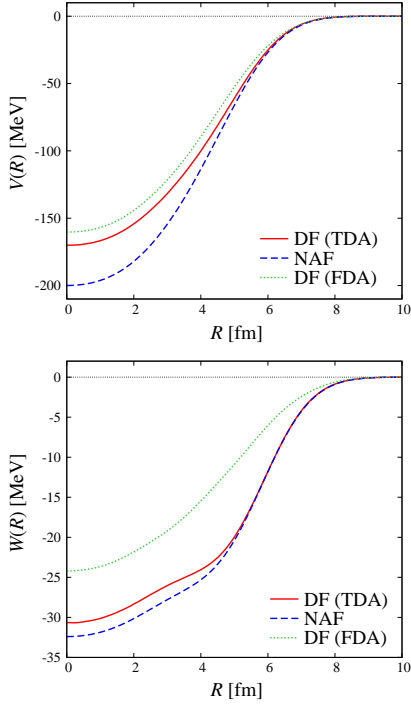


FIG. 6: (Color online) Optical potentials for ${}^4\text{He}+{}^{58}\text{Ni}$ elastic scattering at $E_{\text{in}}/A_P = 26$ MeV. See Fig. 4 for the definition of lines.

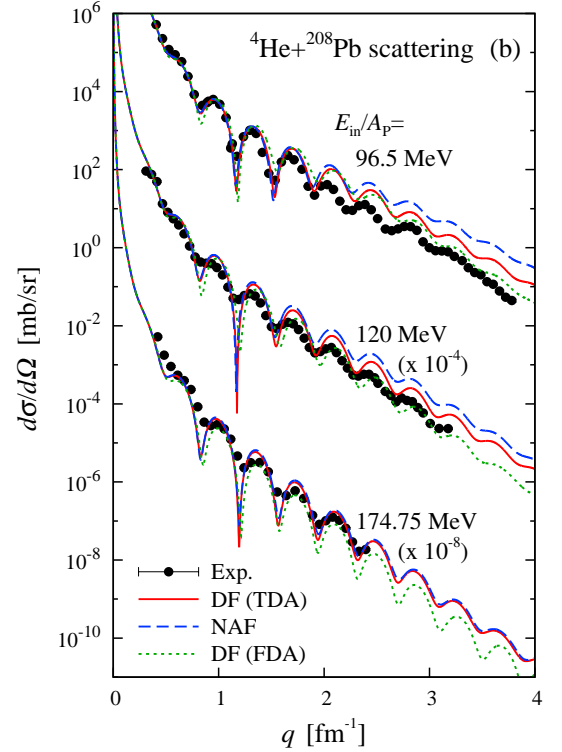
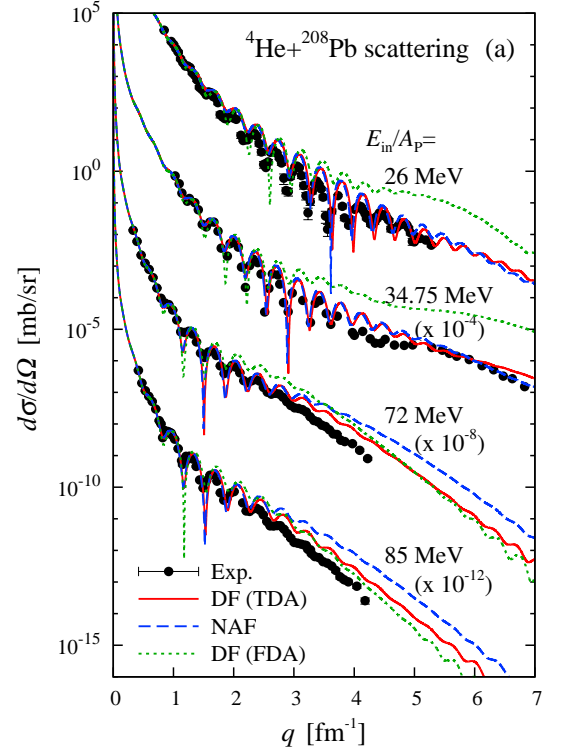


FIG. 7: (Color online) Differential cross sections as a function of transfer momentum q for ${}^4\text{He}+{}^{208}\text{Pb}$ elastic scattering at (a) $E_{\text{in}}/A_P = 26$ –85 MeV and (b) $E_{\text{in}}/A_P = 97$ –175 MeV. The cross section at each E_{in}/A_P is multiplied by the factor shown in the panel. The solid (dotted) line stands the results of the DF-TDA (DF-FDA) model, whereas the dashed line denotes the results of the NAF model. The experimental data are taken from [43, 46–48].

DF-TDA model yields much better agreement with the experimental data than the usual DF model with the frozen-density approximation.

We also investigated the reliability of the nucleon-nucleus folding (NAF) model in which the nucleon-nucleus (NA) potential is folded with the ^4He density. This model is quite practical, since we can use the phenomenological NA optical potential instead of the microscopic NA optical potential. The NAF model well simulates the DF-TDA model for lower energies around $E_{\text{in}}/A_{\text{P}} = 30$ MeV and also for higher energies around

$$E_{\text{in}}/A_{\text{P}} = 175 \text{ MeV}.$$

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- [1] M. Kamimura, M. Yahiro, Y. Iseri, Y. Sakuragi, H. Kameyama, and M. Kawai, *Prog. Theor. Phys. Suppl.* **89**, 1 (1986).
 - [2] N. Austern, Y. Iseri, M. Kamimura, M. Kawai, G. Rawitscher, and M. Yahiro, *Phys. Rep.* **154**, 125 (1987).
 - [3] M. Yahiro, K. Ogata, T. Matsumoto, and K. Minomo, *Prog. Theor. Exp. Phys.* **2012**, 01A206 (2012).
 - [4] G. Bertsch, J. Borysowicz, H. McManus, and W. G. Love, *Nucl. Phys. A* **284**, 399 (1977).
 - [5] J. -P. Jeukenne, A. Lejeune, and C. Mahaux, *Phys. Rev. C* **16**, 80 (1977); J. -P. Jeukenne, A. Lejeune, and C. Mahaux, *Phys. Rep.* **25**, 83 (1976).
 - [6] F. A. Brieva and J. R. Rook, *Nucl. Phys. A* **291**, 299 (1977); *ibid.* 291, 317 (1977); *ibid.* 297, 206 (1978).
 - [7] G. R. Satchler and W. G. Love, *Phys. Rep.* **55**, 183 (1979).
 - [8] G. R. Satchler, "Direct Nuclear Reactions", Oxford University Press, (1983).
 - [9] N. Yamaguchi, S. Nagata, and T. Matsuda, *Prog. Theor. Phys.* **70**, 459 (1983); N. Yamaguchi, S. Nagata, and J. Michiyama, *Prog. Theor. Phys.* **76**, 1289 (1986).
 - [10] L. Rikus, K. Nakano, and H. V. Von Geramb, *Nucl. Phys. A* **414**, 413 (1984); L. Rikus, and H. V. Von Geramb, *Nucl. Phys. A* **426**, 496 (1984).
 - [11] K. Amos, P. J. Dortmans, H. V. Von Geramb, S. Karataglidis, and J. Raynal, in *Advances in Nuclear Physics*, edited by J. W. Negele and E. Vogt (Plenum, New York, 2000) Vol. 25, p. 275.
 - [12] T. Furumoto, Y. Sakuragi, and Y. Yamamoto, *Phys. Rev. C* **78**, 044610 (2008); *ibid.*, **79**, 011601(R) (2009); *ibid.*, **80**, 044614 (2009).
 - [13] S. M. Saliem and W. Haider, *J. Phys. G* **28**, 1313 (2002).
 - [14] B. Sinha, *Phys. Rep.* **20**, 1 (1975). B. Sinha and S. A. Moszkowski, *Phys. Lett. B* **81**, 289 (1979).
 - [15] H. F. Arellano, F. A. Brieva, and W. G. Love, *Phys. Rev. C* **52**, 301 (1995).
 - [16] D. T. Khoa, W. von Oertzen, H. G. Bohlen, and S. Ohkubo, *J. Phys. G* **34**, R111 (2007).
 - [17] T. Furumoto, Y. Sakuragi, and Y. Yamamoto, *Phys. Rev. C* **82**, 044612 (2010).
 - [18] T. Sumi *et al.*, *Phys. Rev. C* **85**, 064613 (2012).
 - [19] K. Minomo, K. Ogata, M. Kohno, Y. R. Shimizu, and M. Yahiro, *J. Phys. G* **37**, 085011 (2010) [arXiv:0911.1184 [nucl-th]].
 - [20] K. Hagino, T. Takehi, and N. Takigawa, *Phys. Rev. C* **74** (2006), 037601.
 - [21] M. Toyokawa, K. Minomo, and M. Yahiro, *Phys. Rev. C* **88**, 054602 (2013).
 - [22] A. J. Koning and J. P. Delaroche, *Nucl. Phys. A* **713** 231 (2003).
 - [23] S. Hama, B. C. Clark, E. D. Cooper, H. S. Sherif, and R. L. Mercer, *Phys. Rev. C* **41**, 2737 (1990).
 - [24] E. D. Cooper, S. Hama, B. C. Clark, and R. L. Mercer, *Phys. Rev. C* **47**, 297 (1993).
 - [25] C. M. Perey and F. G. Perey, *At. Data Nucl. Data Tables* **17**, 1 (1976).
 - [26] K. M. Watson, *Phys. Rev.* **89**, 575 (1953).
 - [27] A. K. Kerman, H. McManus, and R. M. Thaler, *Ann. Phys.* **8**, 551 (1959).
 - [28] M. Yahiro, K. Minomo, K. Ogata, and M. Kawai, *Prog. Theor. Phys.* **120**, 767 (2008).
 - [29] T. Furumoto and Y. Sakuragi, *Phys. Rev. C* **74**, 034606 (2006).
 - [30] K. Minomo, T. Sumi, M. Kimura, K. Ogata, Y. R. Shimizu, and M. Yahiro, *Phys. Rev. C* **84**, 034602 (2011).
 - [31] K. Minomo, T. Sumi, M. Kimura, K. Ogata, Y. R. Shimizu, and M. Yahiro, *Phys. Rev. Lett.* **108**, 052503 (2012).
 - [32] M. Kimura and H. Horiuchi, *Prog. Theor. Phys.* **111**, 841 (2004).
 - [33] M. Kimura, *Phys. Rev. C* **75**, 041302(R) (2007).
 - [34] J. F. Berger, M. Girod, and D. Gogny, *Comput. Phys. Commun.* **63**, 365 (1991).
 - [35] G. Takeda and K. M. Watson, *Phys. Rev.* **97**, 1336(1955).
 - [36] A. Picklesimer and R. M. Thaler, *Phys. Rev. C* **23**, 42(1981).
 - [37] H. de Vries, C. W. de Jager, and C. de Vries, *At. Data Nucl. Data Tables* **36**, 495 (1987).
 - [38] R. P. Singhal, M. W. S. Macauley, and P. K. A. De Witt Huberts, *Nucl. Instr. and Meth.* **148**, 113 (1978).
 - [39] H. H. Chang, B. W. Ridley, T. H. Braid, T. W. Conlon, E. F. Gibson, and N. S. P. King, *Nucl. Phys. A* **270**, 413 (1976).
 - [40] H. Rebel, R. Löhken, G. W. Schweimer, G. Achatz, and G. Hauser, *Z. Phys.* **256**, 258 (1972).
 - [41] J. Albiński *et al.*, *Nucl. Phys. A* **445**, 477 (1985).
 - [42] Y. -W. Lui, D. H. Youngblood, H. L. Clark, Y. Tokimoto, and B. John, *Phys. Rev. C* **73**, 014314 (2006).
 - [43] B. Bonin *et al.*, *Nucl. Phys. A* **445**, 381 (1985).
 - [44] B. K. Nayak *et al.*, *Phys. Lett. B* **637**, 43 (2006).
 - [45] A. Ingemarsson *et al.*, *Nucl. Phys. A* **676**, 3 (2000).
 - [46] G. Hauser, R. Löhken, H. Rebel, G. Schatz, G. W. Schweimer, and J. Specht, *Nucl. Phys. A* **128**, 81

- (1969).
- [47] D. A. Goldberg, S. M. Smith, H. G. Pugh, P. G. Roos, and N. S. Wall, Phys. Rev. C **7**, 1938 (1973).
- [48] M. Uchida *et al.*, Phys. Rev. C **69**, 051301(R) (2004).